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Perturbation Analysis and Design Equations for Open- and Closed-Ring Microstrip Resonators

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Abstract —**Simple closed-form expressions for the resonant frequency and electromagnetic field distribution for various modes of the open- and closed-ring microstrip resonators are derived by utilizing the perturbation analysis of the equivalent curved waveguide model. These results are shown to be in good agreement with the exactly computed values obtained by the solution of the eigenvalue equation for the equivalent waveguide model and the experimental data. The effect of gap capacitance on the eigenvalues of the open-ring resonator is also examined.**

I. Introduction

MICROSTRIP annular ring resonators have been used in recent years for various applications including microwave filters and planar antenna elements [1]–[9]. The basic properties of these structures, that is, the resonant frequency and the field distribution for various modes, have been evaluated by utilizing a number of techniques including the numerical solution of the eigenvalue problem associated with the equivalent two-dimensional curved waveguide model [1]–[9]. Closed-form solutions expressing the resonant frequencies and fields in terms of the geometry of the structure (or the corresponding model) are not yet available for the design of such structures except for the simplified case where the effect of the curvature is totally neglected. In this paper, simple closed-form expressions for the resonant frequencies and the electromagnetic fields are derived by utilizing the perturbation analysis of the equivalent curved waveguide [10], [11] with electric and magnetic walls. The accuracy and range of validity of the results are also examined together with the effects of small gap angles on the resonant characteristics of the open-ring structures.

II. Theory

The magnetic wall curved waveguide models for the open- and closed-ring microstrip resonators are shown in Fig. 1. The model is characterized by its effective dimensions and the medium permittivity which are determined from the solution of the corresponding microstripline problem [12], and the inclusion of the effect of curvature on the model [3], [4]. The model assumes that the substrate height h is small ($h \ll \lambda$, the wavelength) and, hence, the fields are constant along the z-direction. The solutions of interest for fields are then the TM modes with respect to the

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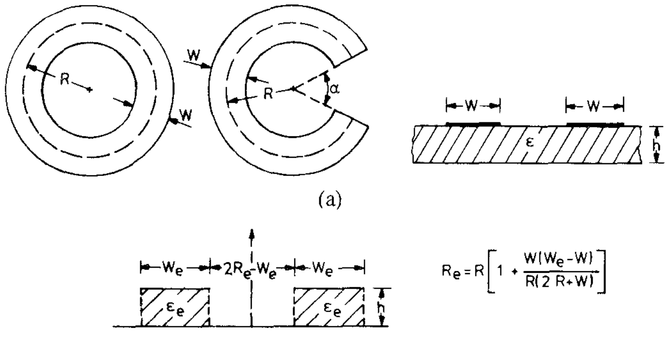

Fig. 1. (a) The microstrip ring resonators. (b) Cross-sectional view of the waveguide model. —— Electric walls. --- Magnetic walls.

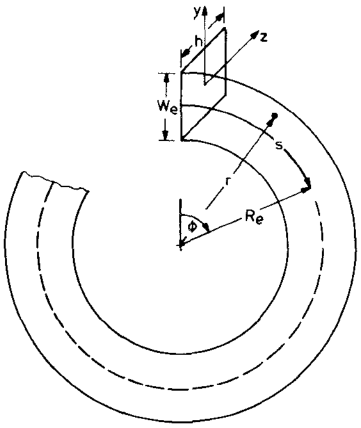

Fig. 2. The curved coordinate system for the model.

direction z. The two-dimensional wave equation for E_z and the boundary conditions in the curved orthogonal coordinate system as characterized by $u_1 = y$, $u_2 = z$, and $u_3 = s = R_e\phi$ (Fig. 2) with corresponding metric coefficients $h_1 = h_2 = 1$ and $h_3 = 1 + y/R_e$ given as [10]

$$\left(1+\frac{y}{R_e}\right)^2\frac{\partial^2 E_z}{\partial y^2}+\frac{1}{R_e}\left(1+\frac{y}{R_e}\right)\frac{\partial E_z}{\partial y}+\frac{\partial^2 E_z}{\partial s^2}+k^2\left(1+\frac{y}{R_e}\right)^2 E_z=0 \quad (1)$$

$$\frac{\partial E_z}{\partial y}=0 \text{ at } y=\pm\frac{W_e}{2}. \quad (2)$$

E_z is a single-valued function of ϕ or s for closed-ring resonator or

$$E_z(y,\phi)=E_z(y,\phi+2\pi) \quad (3)$$

$$\frac{\partial E_z}{\partial\phi \text{ or } \partial s}=0 \text{ at } \phi=\pm\frac{\alpha}{2} \text{ for open-ring resonator.} \quad (4)$$

In the above equations, W_e and R_e are the effective width and radius of the ring, respectively, and α is the gap angle.

A perturbation solution for the above boundary value problem can be found by expanding E_z and the propagation constants β along s in a power series in curvature radius as shown in [10] for a curved rectangular waveguide with electric walls. That is, we expand E_z and β as

$$E_z = Ae^{-j\beta s}\left[E_{z0}+\frac{E_{z1}}{R_e}+\frac{E_{z1}}{R_e^2}+\cdots\right] \quad (5a)$$

$$\beta^2=\beta_0^2\left[1+\frac{B_1}{R_e}+\frac{B_2}{R_e^2}+\cdots\right] \quad (5b)$$

where E_{z0} and β_0 are the solution for the straight waveguide problem ($R\to\infty$), $E_{z1}, E_{z2},\cdots$, are the expansion functions for a given mode, and $B_1, B_2,\cdots$, are constants of expansion of β in the power series. The solutions for the corresponding straight microstrip problem ($R\to\infty$) for various modes are

$$E_{z0}=\cos\left[\frac{p\pi}{W_e}\left(y-\frac{W_e}{2}\right)\right]$$

$$\beta_0^2=k_e^2-\frac{p^2\pi^2}{W_e^2}, \qquad p=0,1,2,\cdots. \quad (6)$$

The boundary conditions along s as given by (3) and (4) imply that

$$\beta R_e = q \qquad \text{for the closed-ring resonator}$$

$$\beta R_e=\frac{q\pi}{(2\pi-\alpha)} \qquad \text{for the open-ring resonator} \quad (7)$$

with $q = 0,1,2\cdots$.

Substitution of (5) into (1) and comparing the like powers of R_e leads to the solution for the expansion functions $E_1, E_{z2},\cdots$, and constants $B_1, B_2,\cdots$, as shown by Lewin *et al.* [10]. The degree of accuracy and the order of complexity of the expressions obviously depends on the number of higher order terms.

The first-order solution for the eigenvalues and fields which includes only one nonzero higher order term in the expansion are given by the following expressions for various TM_{mn0} modes. For the TM_{m00} modes with ideal gap $\alpha\to 0$ (i.e., an open-ring resonator with an infinitesimal gap represented by a magnetic wall), we get

$$k_eR_e=\left[1.25\frac{12-\xi^2}{\xi^4}\left(\sqrt{1+\frac{96}{5}\frac{m^2\xi^4}{(12-\xi^2)^2}}-1\right)\right]^{1/2} \quad (8)$$

and

$$E_z=A\left\{1+k_w^2R_e^2\xi^4\zeta\left[\frac{1}{4}-\frac{\zeta^2}{3}\right]\right\}\cdot\cos\left[\frac{m\pi}{2}\frac{(2\phi-\alpha)}{(2\pi-\alpha)}\right] \quad (9)$$

for any α for all TM_{m00} modes. Here $\xi = W_e/R_e$ is the normalized width, and $\zeta = r - R_e/W_e$ varies from $-1/2$ to $+1/2$ as we go from the inner to the outer radius. For the higher order TM_{mn0} modes, the resonant frequencies are given by

$$k_eR_e=\left[\frac{B-\sqrt{B^2-4AC}}{2A}\right]^{1/2}, \qquad n\geqslant 1 \quad (10)$$

where

$$A = \frac{21 + n^2\pi^2}{12n^4\pi^4}\xi^4$$

$$B = \left[1 + \frac{\xi^2}{12n^2\pi^2}(12 - n^2\pi^2)\right]$$

$$C = \left(\frac{m}{2}\right)^2 + n^2\pi^2\left(\frac{6-\xi^2}{6\xi^2}\right).$$

The first-order perturbation solution for E_z is found to be

$$E_z = A\left\{E_{z0}\left[1 - k_e\frac{y}{R_e}\frac{W_e^2}{2\pi^2n^2}\right] + \frac{W_e^2}{R_e2\pi^2n^2}\frac{\partial E_{z0}}{\partial y}\right.$$

$$\left.\cdot\left[\beta_0^2\left(y^2 - \frac{W_e^2}{4}\right) - \frac{k_e^2W_e^2}{n^2\pi^2}\right]\right\}\cos\left[\frac{m\pi}{2}\frac{(2\phi-\alpha)}{(2\pi-\alpha)}\right] \quad (11)$$

where E_{z0} and β_0 are given by (5) and $y = r - R_e$.

The resonance frequencies of open-ring resonators with finite gap angle α are given by

$$k_eR_e = (k_eR_e)|_{\alpha\to 0}\cdot\frac{2\pi}{(2\pi-\alpha)} \quad (12)$$

where $(k_eR_e)|_{\alpha\to 0}$ is given by (8) and (10).

The integer m in the above equations only assumes even values including zero for the case of closed-ring structures. In general, the above equations (8) and (10) lead to the normalized resonance frequencies for the case of an ideal gap with $\alpha \to 0$. There are twice as many modes for the open-ring resonators than for the closed-rings. These modes correspond to even and odd symmetry with respect to the axis of the gap. The eigenvalues of the closed-ring resonaters corresponds to even modes only, i.e., even modes of open-ring structures with respect to the axis of symmetry.

III. Results

The normalized resonance frequencies for a given structure can be evaluated from (8) and (10) for axial, radial, and higher order modes. For TM_{m00} nodes, these are plotted in Fig. 3 as a function of the ratio of the effective width to the effective radius together with the computed values for the two-dimensional model having an ideal gap with angle $\alpha \to 0$. The open-ring structure supports both the even and the odd modes with respect to the axis of the gap, whereas the closed-ring structure supports solutions corresponding to even values of m only. As seen from the figure, the perturbation expression does lead to fairly accurate results for the resonance frequencies. The accuracy depends on the curvature and is seen to deteriorate for higher order modes for larger W_e/R_e. For example, the perturbation solutions are within 2 percent of the exactly computed values for the fundamental mode with $m=1$ and $n=0$ for W_e/R_e as high as 1.8 and up to the sixth axial-quasi TEM mode for $W_e/R_e=1$. For higher order TM_{mn0} modes, the perturbation solutions as given by (10) do not exist for a given n for a range of W_e/R_e because of the quadratic nature of the equation for the eigenvalues. The solution is found to be very close to the computed

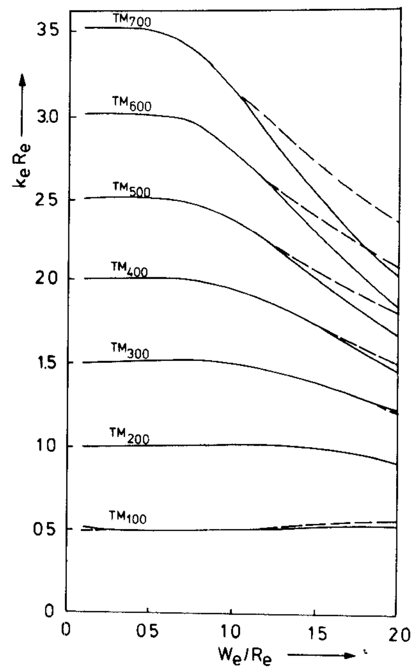

Fig. 3. Normalized resonance frequencies for the quasi-TEM axial (TM_{m00}) modes as a function of the normalized curvature W_e/R_e for an idealized gap with $\alpha \to 0°$. —— Perturbation solutions. --- Solutions of the exact eigenvalue problem.

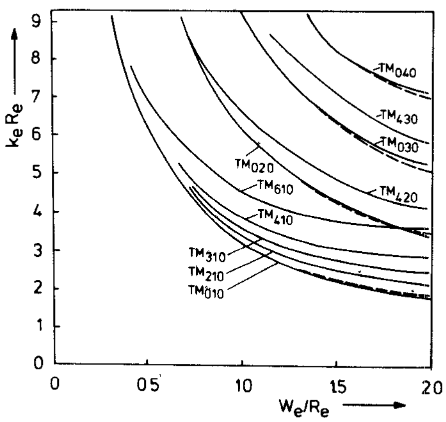

Fig. 4. Normalized resonance frequency of the higher order TM_{mn0} modes. Dashed curves represent the solution of the eigenvalue equation for TM_{0n0} modes.

values over the range of W_e/R_e where $B^2 \geq 4AC$. In Fig. 4, the normalized resonant frequencies for various modes are plotted as a function of the ratio of width to radius based on a simplified expression derived from (10) for all $n \neq 1$ and a semi-empirical formula for $n=1$. This simplified expression, which does lead to solutions for all values of n and m for the whole range of W_e/R_e, is found to be

$$k_eR_e = \sqrt{\left[\left(\frac{m}{2}\right)^2 + \left(\frac{n\pi}{\xi}\right)^2\right]\left[1 + \frac{\xi^2}{12}\left|\left(1 - \frac{12}{n^2\pi^2}\right)\right|\right]}. \quad (13)$$

The exact solution for the corresponding eigenvalue problem for TM_{0n0} modes are also shown in the figure to

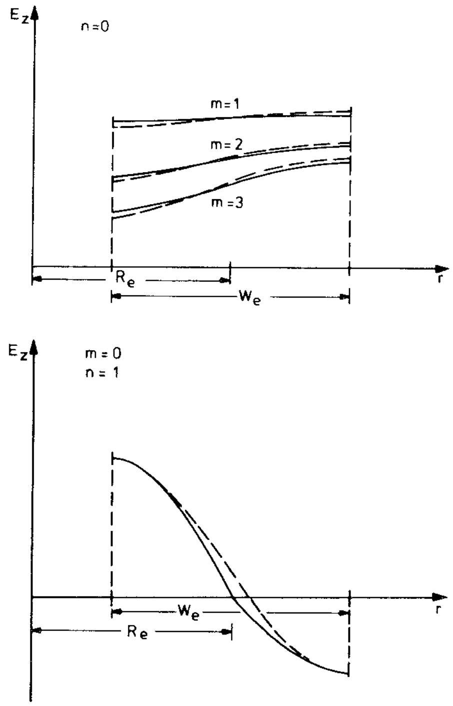

Fig 5. Normalized electric-field variation across the ring for typical lower order modes for $W_e/R_e = 1.2$. —— Perturbation solution. --- Exact solution for the model.

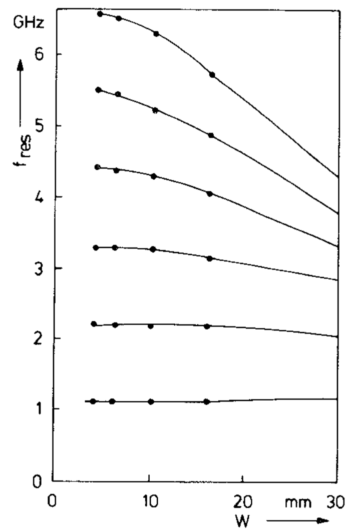

Fig. 6 Resonance frequencies versus width for a given ring radius and the gap angle. $\epsilon_r = 2.23$, $h = 0.79$ mm, $R = 16$ mm, $\alpha = 15°$. Solid points • represent measured values.

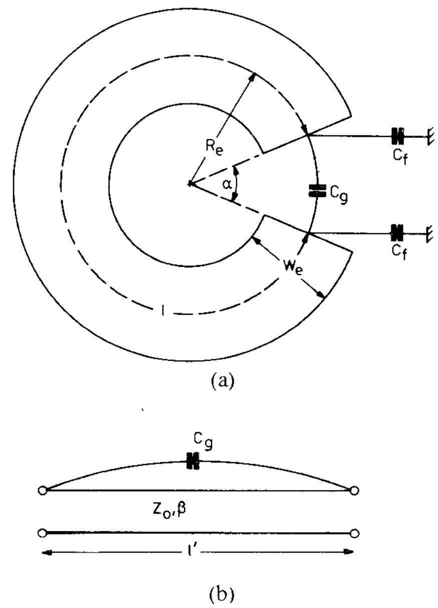

Fig 7 (a) A model for including the stray fields in the gap. (b) The equivalent transmission-line model.

demonstrate that the perturbation solution of (13) is indeed close to the computed values for these higher order modes.

The variation of the normalized electric field as given by (9) and (11) is shown in Fig. 5. Again, the computed values found from the solution of the exact eigenvalue problem associated with the magnetic wall model in cylindrical coordinate systems are plotted on the same figures for comparison. The same statements regarding the accuracy of the perturbation solutions hold here as those made for the eigenvalues. The magnetic-field components are easily found in terms of E_z from the Maxwell's equation. It should be noted that these field distributions correspond to the solution of the magnetic wall model boundary value problem and not the real structure. The solutions do, however, provide a measure of the actual field and can be useful in estimating certain parameters, such as the resonator Q and radiation characteristics.

The perturbation solutions have been compared with the exact solution corresponding to the magnetic wall model since the validity of the latter has been established from various experimental results [1]–[4], [9] for both closed- and open-ring resonators. For example, Fig. 6 shows the resonant frequencies calculated from (8) and (12) and the measured values for a set of open-ring resonators with $\alpha = 15°$. It should be mentioned that these perturbation solutions for eigenvalues and fields do lead to a significant improvement in the accuracy of the results as compared to the solutions corresponding to the straight waveguide model.

The effect of gap capacitance for the open-ring resonators has been experimentally seen to be not very significant unless the gap angle is small [9]. For modes corresponding to $n = 0$, i.e., the quasi-TEM modes on the resonator, the effect of the gap can be estimated by considering the line and the gap equivalent circuit shown in Fig. 7(a). The stray field at the end of the even-mode capacitance represented by C_f can be easily included in the analysis by defining an effective length as

$$l' = (2\pi - \alpha) R_e + 2\frac{C_f Z_0}{\sqrt{\mu_0 \epsilon_e}}. \tag{14}$$

The effect of gap capacitance C_g is then estimated by considering the resonator line having phase constant β, characteristic impedance Z_0 connected via C_g as shown in Fig. 7(b). It is easily seen that the resonance frequency

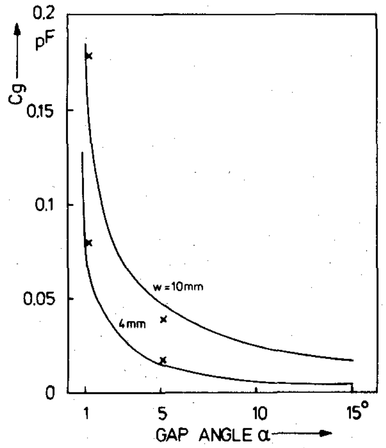

Fig. 8. Gap capacitance C_g as a function of the gap angle for two typical cases. $\epsilon_r = 2.23$, $h = 0.79$ mm, $R = 16.0$ mm, × represents values computed from the experimental data.

must then be a solution of

$$\sin \beta l' = 2\omega C_g Z_0 (1 - \cos \beta l'). \tag{15}$$

Equation (15) is satisfied for two possible conditions:

$$\beta l' = 2n\pi, \qquad n \text{ is an integer} \tag{16a}$$

or

$$\cot \frac{\beta l'}{2} = 2\omega C_g Z_0. \tag{16b}$$

Equations (16a) and (16b) lead to even- and odd-mode resonances, respectively. As seen from these equations, and as should be expected from the field distribution, the gap capacitance C_g has no effect on the even modes. These are the only modes excited on closed annular rings. Whereas for odd modes, the resonance frequency is decreased as shown by (16b). The smaller the gap angle, the larger the gap capacitance, and in the limit $\alpha \to 0, C_g \to \infty$ and only the even modes can exist since, for this case, (16b) degenerates into (16a).

C_g and C_f can be found numerically by solving for the static capacitance of the closed-ring and the capacitance matrix of an open-ring structure with two gaps. The behavior of C_g as a function of the gap angle computed by utilizing a straightforward finite-element program is illustrated in Fig. 8 for two cases of the ring widths. This C_g was also evaluated from the experimental data for the resonance frequencies for the even and odd modes for these cases from (16a) and (16b) in order to confirm the validity of this model. These values are also shown in Fig. 8 and are seen to agree well with the computed values. The effect of gap capacitance is not very significant unless the gap angle is less than about 5°. However, this gap capacitance is dependent on the gap angle and substrate dielectric constant and height for given ring dimensions, and it can also be controlled or increased by loading the gap with a capacitive element such as an interdigital capacitor. This effect of gap capacitance on the resonance frequency of the fundamental mode can be utilized to design compact resonators or resonators with desired physical dimensions.

IV. Concluding Remarks

Simple closed-form expressions for the resonant frequencies and field distribution for various modes of the open- and closed-ring microstrip resonators have been derived by utilizing the perturbation analysis of the two-dimensional magnetic wall curved waveguide model. These expressions together with the solution for the parameters of the waveguide model for curved microstrips can be used directly to design these resonators for various applications as microwave circuit elements and also to help study their radiation characteristics. The results have been shown to be quite accurate, except when the ring width is near its limiting value where the ring degenerates into a disc.

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Characteristics of Metal–Insulator–Semiconductor Coplanar Waveguides for Monolithic Microwave Circuits

ROBERTO SORRENTINO, MEMBER, IEEE, GIORGIO LEUZZI, AND AGNÈS SILBERMANN

Abstract—**Using a full-wave mode-matching technique, an extensive analysis is presented of the slow-wave factor, attenuation, and characteristic impedance of a metal–insulator–semiconductor coplanar waveguide (MISCPW) as functions of the various structural parameters. Design criteria are given for low-attenuation slow-wave propagation. By a proper optimization of the structure, performances comparable with or even better than those of alternative structures proposed in the literature are theoretically predicted.**

I. Introduction

MONOLITHIC MICROWAVE integrated circuits, using both Si and GaAs technologies, have an increasing impact in a number of applications because of higher reliability, reproducibility, and potentially lower costs [1]. It has already been pointed out that accurate analysis techniques are required in order to reduce necessity for trimming, which is more difficult than for hybrid integrated circuits. Even in this case, however, full-wave analyses are necessary to study propagation effects in active devices [2]. Gigabit logic is another area where propagation effects have to be accounted for through the use of accurate theoretical analyses [3].

Slow-wave propagation in metal–insulator–semiconductor and Schottky-contact planar transmission lines has been both experimentally observed and theoretically explained from different points of view [3]–[10]. The slow-wave properties of such transmission lines can be used to reduce the dimensions and cost of distributed elements to realize delay lines or, when Schottky-contact lines are used, for variable phase shifters, voltage-tunable filters, etc.

A drawback of these slow-wave structures is the loss associated with the semiconducting layer. As an example, the GaAs metal–insulator–semiconductor coplanar waveguide (MISCPW) experimented by Hasegawa and his coworkers [6], [11] presented an attenuation greater than 1 dB/mm, with a slowing factor of about 30 at the frequency of 1 GHz. Since losses and slow-wave effects depend on the distribution of the electromagnetic field inside the various regions of the structure, accurate analyses are required to determine the most favorable conditions for the practical use of such transmission lines.

An extensive study of the properties of MISCPW, based on a full-wave technique, is presented in this paper. The influence of the various structural parameters on the characteristics of the structure is investigated, together with the effect of the addition of a back conducting plane, which

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